

# Modeling of Hybrid Growth Wastewater Bio-reactor

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## **Abstract**

The attached/suspended growth mixed reactors are considered one of the recently tried approaches to improve the performance of the biological treatment by increasing the volume of the accumulated biomass in terms of attached growth as well as suspended growth. Moreover, the domestic WW can be easily mixed with a high strength non-hazardous industrial wastewater and treated together in these bio-reactors if the need arises. Modeling of Hybrid hybrid growth wastewater reactor addresses the need of understanding the rational of such system in order to achieve better design and operation parameters. This paper aims at developing a heterogeneous mathematical model for hybrid growth system considering the effect of diffusion, external mass transfer, and power input to the system in a rational manner. The model will be based on distinguishing between liquid/solid phase (bio-film and bio-floc). This model would be a step ahead to the fine tuning the design of hybrid systems based on the experimental data of a pilot plant to be implemented in near future.

**Keywords:** Activated sludge AS, hybrid reactor, upgrading wastewater systems, Modeling, Settling tanks.

## **1. Modeling of Wastewater Hybrid Bio-Reactors:**

Efforts have been made to improve the performance of activated sludge systems by introducing support media into aeration basins thus combining suspended and fixed growth processes. The results reported are general and conclude that the combination of suspended microorganisms and fixed film enhances BOD removal and solid settling (Bolte and Hill, 1985; Lee, 1992; Dertil et al., 1992; Polprasert and Agarwalla, 1994; vanLoosdrecht et al., 1995; Debus, 1995; Welander et al., 1998; Gebara, 1999; Ayoub et al., 2003)

Plastic nets are fitted vertically inside the aeration tank of a conventional activated sludge process in a laboratory scale unit thus making it a hybrid growth reactor combining both suspended and fixed biomass (Gebara, 1999). The addition of nets improved the BOD<sub>5</sub> removal efficiency and sludge settling efficiency considerably. A theoretical mathematical steady state model predicting the performance of the hybrid growth reactor is proposed and the accuracy is affirmed by comparison with experimentally measured results. The hybrid growth model proposed in this paper considers the aeration tank to be composed of two reactors in series. The first reactor with nets (attached growth) is followed by the second reactor which comprises the bulk water volume (suspended growth).

Another hybrid reactor was characterized by a competition for substrates between two growth-type bacteria (attached and suspended), which can not be accurately predicted by single-growth models (Lee, 1992). This paper presents a model that considers two growths competing for a single substrate in such a completely mixed reactor at steady-state condition. The critical advantage of this model stems from the fact that it maintains all essential concepts of single-

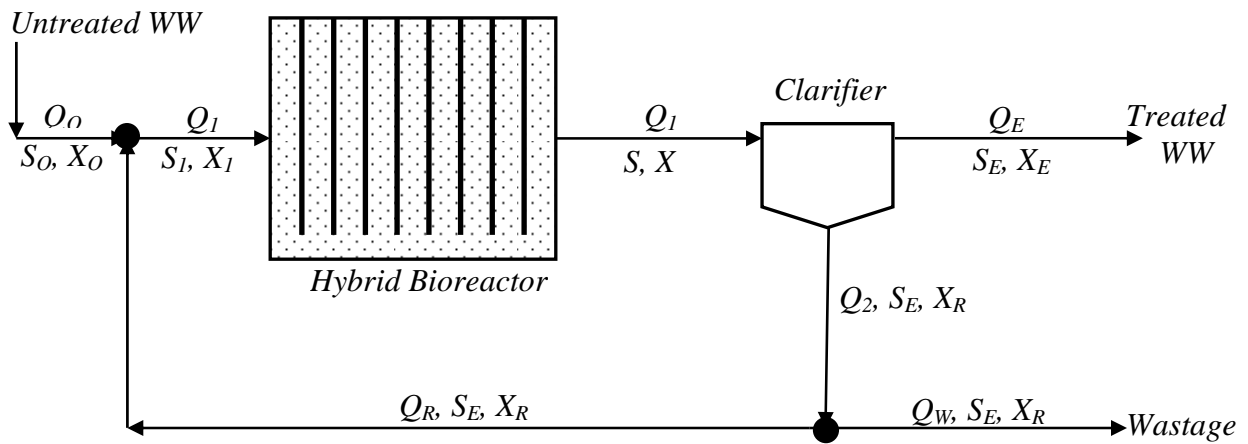
growth kinetics but uses only two fundamental parameters of empty-bed hydraulic retention time and suspended biomass solid retention time to predict the competing results. These predictions are very useful for analyzing the design and performance of a variety of hybrid reactors that mix completely.

An important feature of this work is that, while describing the biomass balance for suspended growth, the loss of biofilm has been considered. This loss of biomass from biofilm consists of shear losses and biomass decay.

The kinetic steady state model presented in another work adequately predicts the reactor performance for both anaerobic suspended particle – attached growth reactors and anaerobic fixed bed reactors under a variety of operating conditions (Bolte and Hill, 1985). Some of the assumptions are: completely mixed (i.e. homogeneous) conditions exist within the reactor, which is not true for attached growth systems which rely on non-homogeneity to enhance process kinetics. However, for suspended particle – attached growth reactors where mixing occurs, the bulk liquid substrate concentration can be considered reasonably uniform throughout the reactor volume. Another assumption is that the biological solids retention time is large relative to the hydraulic retention time, such that the ratio between hydraulic and biological solids retention times was negligible. This strips the model of the ability to distinguish between media types.

Figure 1 shows the schematic of the proposed hybrid growth bio-reactor. Untreated WW after undergoing primary treatment is mixed with the activated sludge and then enters the hybrid bioreactor. The bioreactor has plastic nets which act as support for the attached biofilm growth. Remaining volume of the bioreactor has microorganisms in the suspended form. It is assumed here that

there is no concentration gradient along the suspended phase of the bioreactor. Moreover, all the nets have identical characteristics. The product stream from the bioreactor is fed to a clarifier, which separates the solids from the liquid. The clear liquid is collected at the top of clarifier and forms the treated WW stream. A part of the solids collected from the bottom of the clarifier is recycled back to the bioreactor as activated sludge and the remaining is discarded as waste. It is assumed that no biological reactions occur within the clarifier.



**Figure 1: Schematic of the Hybrid wastewater treatment process**

## **2. Formulation of the Dynamic Model:**

Mass balances are required for each of the equipments and junctions to formulate the flow rates and concentrations. These are presented below:

## Mixer

*Flow*

$$Q_O + Q_R = Q_1$$

$$\Rightarrow Q_1 = Q_O + Q_R \quad (1)$$

*Substrate*

$$Q_O S_O + Q_R S_E = Q_1 S_1$$

$$\Rightarrow S_1 = \frac{Q_O S_O + Q_R S_E}{Q_1} \quad (2)$$

*Biomass*

$$Q_O X_O + Q_R X_R = Q_1 X_1$$

$$\Rightarrow X_1 = \frac{Q_O X_O + Q_R X_R}{Q_1} \quad (3)$$

## Clarifier

*Flow*

$$Q_E + Q_2 = Q_1$$

$$\Rightarrow Q_2 = Q_1 - Q_E \quad (4)$$

*Substrate*

$$Q_1 S = Q_E S_E + Q_2 S_E$$

$$\Rightarrow S_E = S \quad (5)$$

*Biomass*

$$Q_1 X = Q_E X_E + Q_2 X_R$$

$$\Rightarrow X_R = \frac{Q_1 X - Q_E X_E}{Q_2} \quad (6)$$

## Splitter

*Flow*

$$Q_2 = Q_R + Q_W$$

$$\Rightarrow Q_R = Q_2 - Q_W \quad (7)$$

Where,

$$\text{Empty reactor volume} = V \text{ m}^3$$

$$\text{Volume available for suspended media} = V_S \text{ m}^3$$

$$\text{Volume occupied by film and support} = V_F \text{ m}^3$$

$$\text{Volume of the attached phase (film)} = A_F L_F \text{ m}^3$$

$$X = \frac{\text{kg biomass}}{\text{m}^3 \text{ of suspended phase}}$$

$$S = \frac{\text{kg substrate}}{\text{m}^3 \text{ of suspended phase}}$$

$$X_F = \frac{\text{kg biomass}}{\text{m}^3 \text{ of attached phase}}$$

$$S_F = \frac{\text{kg substrate}}{\text{m}^3 \text{ of attached phase}}$$

The following additional parameters have been defined in the model:

$$\text{Recycle ratio (fraction of feed recycled): } \alpha = \frac{Q_R}{Q_O}$$

$$\text{Treatment factor (fraction of wastewater feed treated): } \beta = \frac{Q_E}{Q_O}$$

### **3. Dynamic model equations for the bioreactor:**

Dynamic model of the bioreactor involves equations for separate phases (suspended and attached).

#### **Suspended Phase**

Suspended phase consists of substrate and biomass which are considered to be perfectly mixed. Substrate is consumed by the suspended biomass and also diffuses into the attached film. Biomass growth is related to the substrate consumption through a yield factor. There is a gain in suspended biomass through the breakage of the attached biomass film. Continuous death of the suspended biomass is also considered. The dynamic model for the suspended phase is a set of initial value ordinary differential equations.

*Substrate:*

$$\begin{aligned} V_S \frac{dS}{dt} &= Q_1 (S_1 - S) - \left( \frac{k S X}{K_S + S} \right) V_S - k_m A_F (S - S_F) \\ \Rightarrow \frac{dS}{dt} &= \frac{Q_1}{V_S} (S_1 - S) - \left( \frac{k S X}{K_S + S} \right) - \frac{k_m A_F}{V_S} (S - S_F) \end{aligned} \quad (8)$$

At  $t = 0$ ,  $S = S(0)$

*Biomass:*

$$\begin{aligned} V_S \frac{dX}{dt} &= Q_1 (X_1 - X) + Y_{XS} \left( \frac{k S X}{K_S + S} \right) V_S + k_b X_F A_F L_F - k_{Sd} X V_S \\ \Rightarrow \frac{dX}{dt} &= \frac{Q_1}{V_S} (X_1 - X) + Y_{XS} \left( \frac{k S X}{K_S + S} \right) + \frac{k_b X_F A_F L_F}{V_S} - k_{Sd} X \end{aligned} \quad (9)$$

At  $t = 0$ ,  $X = X(0)$

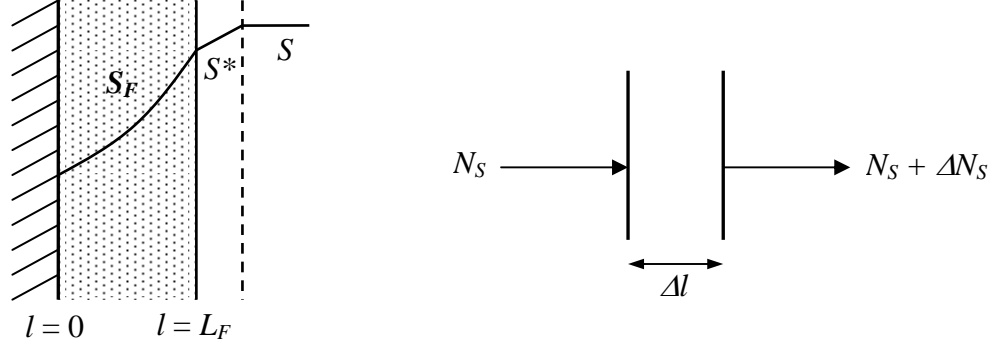
## **Attached Film**

Substrate diffuses into the attached biofilm according to Fick's law. There is a concentration gradient along the thickness of the biofilm, thus making it a distributed system. Dynamic model equations for substrate and biomass in both phases are coupled through the terms involving the diffusion of substrate and the breakage of attached biofilm. It is assumed that the substrate diffused into the biofilm is consumed completely by the biomass (Atkinson and Davies, 1974; Williamson and McCarty, 1976; Rittman and McCarty, 1980a; 1980b; Rittman, 1982a; 1982b; Rittman and Brunner, 1984; Chang and Rittman, 1987a; 1987b; 1988; Saez and Rittman, 1988; Tyagi and Vembu, 1990). The thickness of the biofilm is assumed to be constant throughout the surface area of the support. The dynamic model for substrate in the attached phase is a partial differential equation, which has here been converted into a set of ordinary differential equations through orthogonal collocation technique. The model for the attached biomass is an ordinary differential equation.

### ***Substrate:***

Figure 2 shows the elemental balance on the substrate in attached biofilm, which can be modeled as follows:





**Figure 2: Concentration gradient and elemental balance on the substrate in attached biofilm**

$$\begin{aligned}
 N_S A_F &= (N_S + \Delta N_S) A_F + r(A_F \Delta l) + (A_F \Delta l) \frac{\partial S_F}{\partial t} \\
 \Rightarrow 0 &= \Delta N_S + r \Delta l + \Delta l \frac{\partial S_F}{\partial t} \\
 \Rightarrow \frac{\partial S_F}{\partial t} &= -\frac{\partial N_S}{\partial l} - r
 \end{aligned} \tag{10}$$

From Fick's first law of diffusion:

$$N_S = -D_e \frac{\partial S_F}{\partial l}$$

Thus equation 10 becomes,

$$\Rightarrow \frac{\partial S_F}{\partial t} = D_e \frac{\partial^2 S_F}{\partial l^2} - \left( \frac{k S_F X_F}{K_S + S_F} \right) \tag{11}$$

In terms of dimensionless length  $\left( \omega = \frac{l}{L_F} \right)$ ,

$$\begin{aligned}
 \Rightarrow \frac{\partial S_F}{\partial t} &= \frac{D_e}{L_F^2} \frac{\partial^2 S_F}{\partial (l/L_F)^2} - \left( \frac{k S_F X_F}{K_S + S_F} \right) \\
 \Rightarrow \frac{\partial S_F}{\partial t} &= \frac{D_e}{L_F^2} \frac{\partial^2 S_F}{\partial \omega^2} - \left( \frac{k S_F X_F}{K_S + S_F} \right)
 \end{aligned} \tag{12}$$

Initial condition:

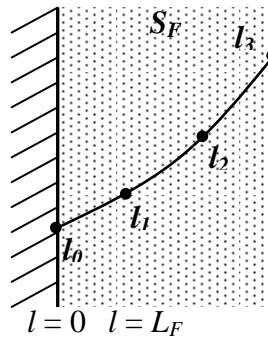
At  $t = 0$ ,  $S_F = S_F(0)$

Boundary conditions:

$$\text{At } \omega = 0, \frac{\partial S_F}{\partial \omega} = 0 \text{ and At } \omega = 1, k_m(S - S_F) = \frac{D_e}{L_F} \frac{\partial S_F}{\partial \omega} \Big|_{\omega=1} \quad (13)$$

### Use of Orthogonal Collocation:

Two internal collocation points are considered for simplifying the partial differential equation. Figure 3 shows the collocation points and Table 3 shows the values of the collocation coefficients obtained from Legendre's polynomials.



**Figure 3: Collocation points**

**Table 3: Values of collocation coefficients using Legendre polynomial**

$i$	$l_i$	$B_{i1}$	$B_{i2}$	$B_{i3}$	$A_{i1}$	$A_{i2}$	$A_{i3}$
0	0						
1	0.285235	- 4.7399	5.6771	- 0.9373			
2	0.765055	8.3229	- 23.2601	14.9373			
3	1.0				1.7915	- 8.7915	7.00

Use of orthogonal collocation method transforms equation 12 into the following set of ordinary differential equations:

$$\frac{dS_{F1}}{dt} = \frac{D_e}{L_F^2} (B_{11} S_{F1} + B_{12} S_{F2} + B_{13} S_{F3}) - \left( \frac{k S_{F1} X_F}{K_S + S_{F1}} \right) \quad (14)$$

$$\frac{dS_{F2}}{dt} = \frac{D_e}{L_F^2} (B_{21} S_{F1} + B_{22} S_{F2} + B_{23} S_{F3}) - \left( \frac{k S_{F2} X_F}{K_S + S_{F2}} \right) \quad (15)$$

The boundary condition (equation 13) for the partial differential equation becomes:

$$A_{31} S_{F1} + A_{32} S_{F2} + A_{33} S_{F3} = \frac{L_F k_m}{D_e} (S - S_{F3}) \quad (16)$$

***Biomass:***

The biomass growth inside the attached biofilm is modeled by considering an average substrate concentration inside the biofilm. The model equation is:

$$\begin{aligned}
A_F L_F \frac{dX_F}{dt} &= Y_{XS} \left( \frac{k \left( \frac{S_{F1} + S_{F2} + S_{F3}}{3} \right) X_F}{K_S + \left( \frac{S_{F1} + S_{F2} + S_{F3}}{3} \right)} \right) A_F L_F - k_b X_F A_F L_F - k_{Fd} X_F A_F L_F \\
\Rightarrow \frac{dX_F}{dt} &= Y_{XS} \left( \frac{k \left( \frac{S_{F1} + S_{F2} + S_{F3}}{3} \right) X_F}{K_S + \left( \frac{S_{F1} + S_{F2} + S_{F3}}{3} \right)} \right) - k_b X_F - k_{Fd} X_F \quad (17)
\end{aligned}$$

At  $t = 0$ ,  $X_F = X_F(0)$

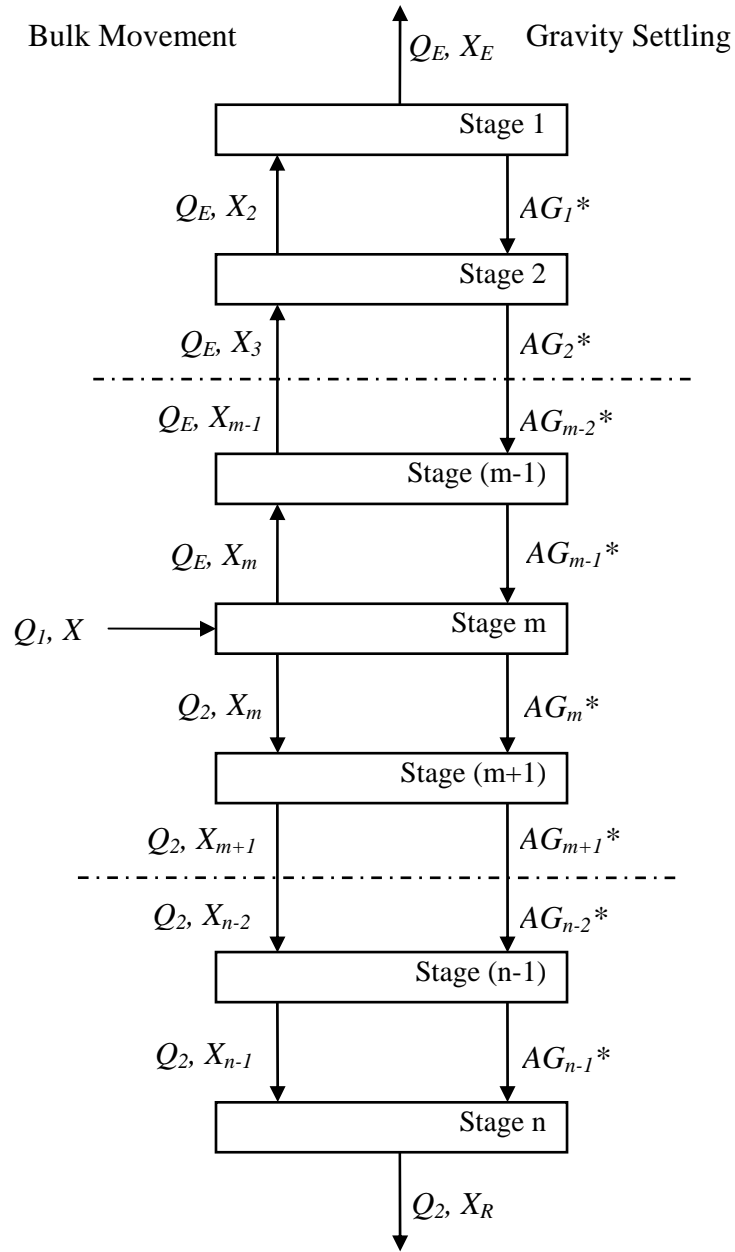
#### **4. Clarifier Modeling:**

The product stream from the bioreactor is fed to a clarifier where the solids are separated from the treated WW. A comprehensive model for a solids-liquid separator requires components for both thickening and clarification. Basically, the clarifier is divided into a number of finite horizontal layers and a material balance is written for each layer assuming complete mixing (Wilson et al., 1980; Tesarik et al., 1986; Haertel and Poepel, 1992; Stamou, 1997; Chancelier et al, 1997a; 1997b; Deininger et al., 1998; Diehl and Jeppsson, 1998; Ellis et al., 1999; Messenger et al., 1999).

As shown in Figure 4, the clarifier is divided into  $n$  layers with the feed entering in layer  $m$ . The layers above the feed zone consist of layers 1 through  $m-1$ . And the layers below the feed zone consist of  $m+1$  through  $n$ . Fluid flows upward from the feed zone at the rate determined by the overflow, and downward at the rate at which thickened underflow is removed. In the region above layer  $m$ , the solids are assumed to have a gravitational settling velocity greater than the upward movement of the fluid in order to be separated from the overflow. A threshold concentration  $X_{thresh}$  is defined in order to describe

behavior in the upper section of the clarifier. Whenever, the solids concentration is greater than  $X_{thresh}$ , the settling flux in that layer affects the rate of settling within adjacent layers. It is presumed that the threshold concentration corresponds to the onset of hindered settling behavior. Following assumptions have been made in formulating the model equations:

1. The continuous clarifier does not exhibit vertical dispersion.
2. The concentration of suspended solids is completely uniform within any horizontal plane within the clarifier.
3. The mass flux into a differential volume can not exceed the mass flux the volume is capable of passing, nor can it exceed the mass flux which the volume immediately below it is capable of passing.
4. The bottom of the clarifier represents a physical boundary to separation and the solids flux due to gravitational settling is zero at the bottom.
5. The gravitational settling velocity is a function only of suspended solids concentration.
6. There is no significant biological reaction affecting solids mass and substrate concentrations within the clarifier.



**Figure 4: Idealized solid-liquid separator**

Where,

$A$  = Cross sectional area of clarifier ( $m^2$ )

$G$  = Gravitational settling flux ( $kg/hr\ m^2$ )

$Q$  = Flow rate ( $m^3/hr$ )

Dynamic equations are formulated for five different regions as shown below:

Top layer ( $i = 1$ )

$$A h \frac{dX_1}{dt} = Q_E X_2 - Q_E X_E - A G_1^*$$
$$\Rightarrow \frac{dX_1}{dt} = \frac{1}{A h} [Q_E X_2 - Q_E X_E - A G_1^*] \quad (18)$$

Feed layer ( $i = m$ )

$$A h \frac{dX_m}{dt} = Q_1 X - Q_2 X_m - Q_E X_m + A G_{m-1}^* - A G_m^*$$
$$\frac{dX_m}{dt} = \frac{1}{A h} [Q_1 X - Q_2 X_m - Q_E X_m + A G_{m-1}^* - A G_m^*] \quad (19)$$

Bottom layer ( $i = n$ )

$$A h \frac{dX_n}{dt} = Q_2 X_{n-1} - Q_2 X_R + A G_{n-1}^*$$
$$\frac{dX_n}{dt} = \frac{1}{A h} [Q_2 X_{n-1} - Q_2 X_R + A G_{n-1}^*] \quad (20)$$

Layers between top and feed layer ( $2 \leq i \leq m-1$ )

$$A h \frac{dX_i}{dt} = Q_E X_{i+1} - Q_E X_i + A G_{i-1}^* - A G_i^*$$

$$\frac{dX_i}{dt} = \frac{1}{Ah} [Q_E X_{i+1} - Q_E X_i + A G_{i-1}^* - A G_i^*] \quad (21)$$

Layers between feed and bottom layer ( $m+1 \leq i \leq n-1$ )

$$Ah \frac{dX_i}{dt} = Q_2 X_{i-1} - Q_2 X_i + A G_{i-1}^* - A G_i^*$$

$$\frac{dX_i}{dt} = \frac{1}{Ah} [Q_2 X_{i-1} - Q_2 X_i + A G_{i-1}^* - A G_i^*] \quad (22)$$

Additional equations:

$G_i = v_{si} X_i$  is the settling flux

Where,

$v_{si} = a e^{-bX_i}$  is the gravitational settling velocity

and  $a, b$  are empirical constants.

For  $2 \leq i \leq m$

$$G_i^* = \begin{cases} G_i & \text{if } X_i \leq X_{thresh} \\ \min(G_i, G_{i+1}) & \text{if } X_i > X_{thresh} \end{cases}$$

For  $m+1 \leq i \leq n$

$$G_i^* = \min(G_i, G_{i+1})$$

The overall hybrid growth WW unit can be simulated by simultaneously solving equations 1 through 22. In the following section, some of the model simulation results are presented.



## 5. Model Simulation Results:

An experimental hybrid growth bioreactor was built and modeled in the work by Gebara, 1999. Area available for attached film growth was varied by changing the number of plastic nets in the experimental setup. He concluded that the WW treatment efficiency and stability can be improved in activated sludge systems by the introduction of attached biofilm. The simulation results of the developed hybrid growth model are compared with the experimental results reported in Gebara, 1999.

**Table 1: Experimental values used in simulation (Gebara, 1999)**

	Values used in experiments and model simulations
$Q_o$ (kg/day)	68.5
$Q_R$ (kg/day)	40.0
a	0.58
$S_o$ (kg/m <sup>3</sup> )	0.48
Support area per net (m <sup>2</sup> )	0.072

Table 1 shows the values of experimental parameters reported in Gebara, 1999. These values were used for solving the model.

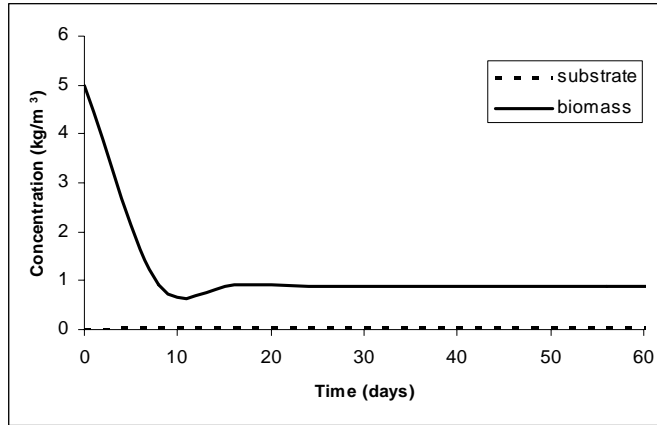
Comparison of the model simulation and experimental results was performed for two different support areas (6 and 24 nets). The results are tabulated in Table 2. It can be seen that the model results correlate well, but are slightly less than the experimental values. It is observed that the experimental and model results generally exhibit the similar trend between the cases. But, the

effluent BOD ( $S_E$ ) calculated from the model does not change with the change in the number of nets, while  $S_E$  drops with an increase in the number of nets in the experiment.

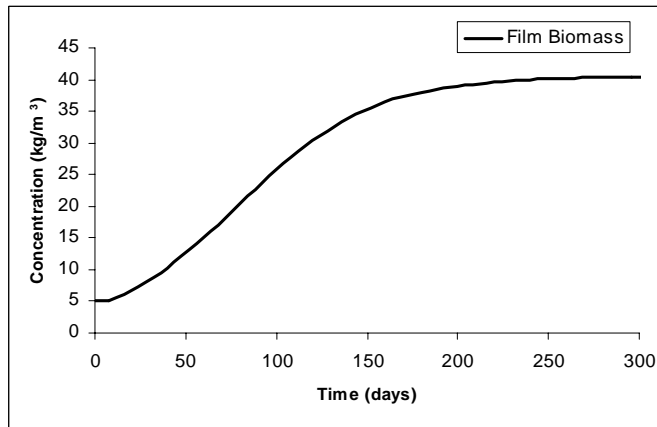
**Table 2: Comparison of experimental and model simulation results for two different support areas (\* Gebara, F., 1999)**

	6 Nets		24 Nets	
	Experimental*	Model	Experimental*	Model
$A_F$ (m <sup>2</sup> )	0.432	0.432	1.728	1.728
$S_E$ (kg/m <sup>3</sup> )	0.046	0.0332	0.02	0.032
$X_E$ (kg/m <sup>3</sup> )	0.005	0.0	0.002	0.0
$X_F$ (kg/m <sup>3</sup> )	54.63	40.53	48.15	19.75
Wastage (kg/day)	0.0262	0.0142	0.0219	0.0143
$X_R$ (kg/m <sup>3</sup> )	3.2	2.085	3.0	2.091

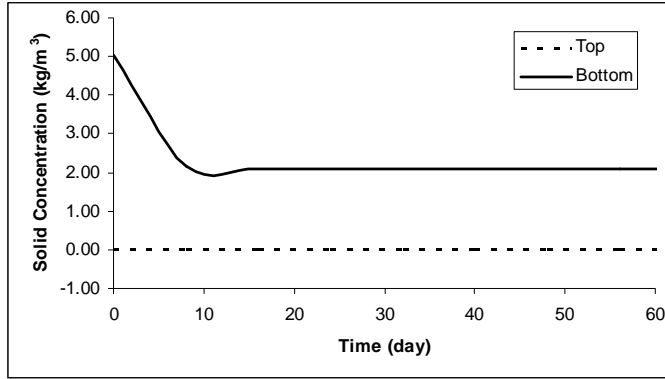
The dynamic profiles of some of the process variables are shown in the following pages. These profiles are for the cases with 6 and 24 nets. Figures 5-7 show the dynamic profiles for the case with 6 plastic nets. Figures 8-10 show the dynamic profiles for the case with 24 nets.



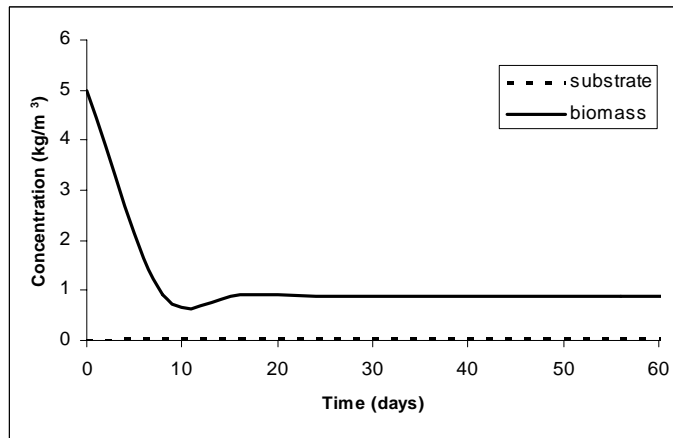
**Figure 5: Dynamic profile of suspended substrate and biomass with 6 nets**



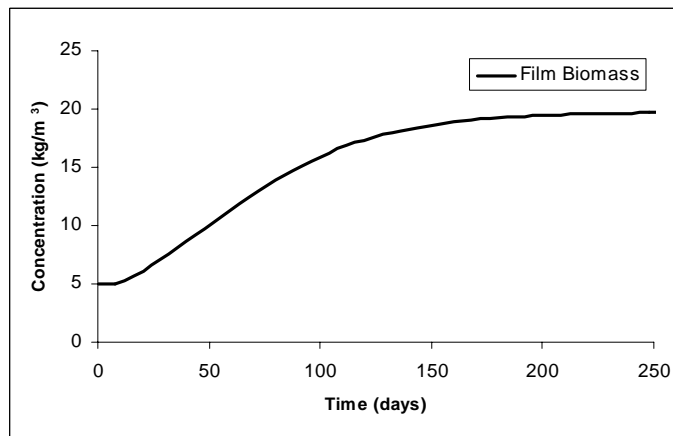
**Figure 6: Dynamic profile of attached biomass with 6 nets**



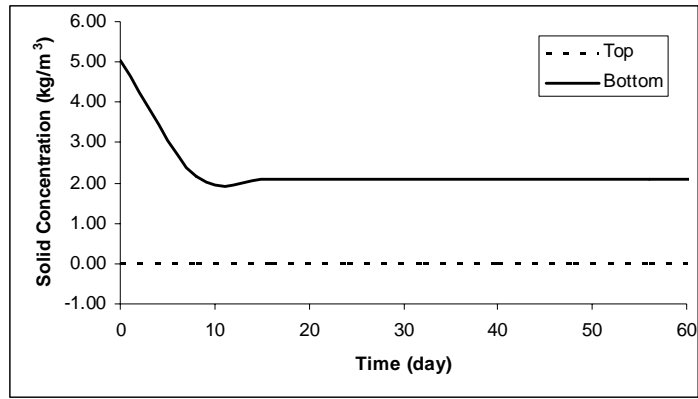
**Figure 7: Dynamic profile of solid concentration in the clarifier for 6 nets**



**Figure 8: Dynamic profile of suspended substrate and biomass with 24 nets**

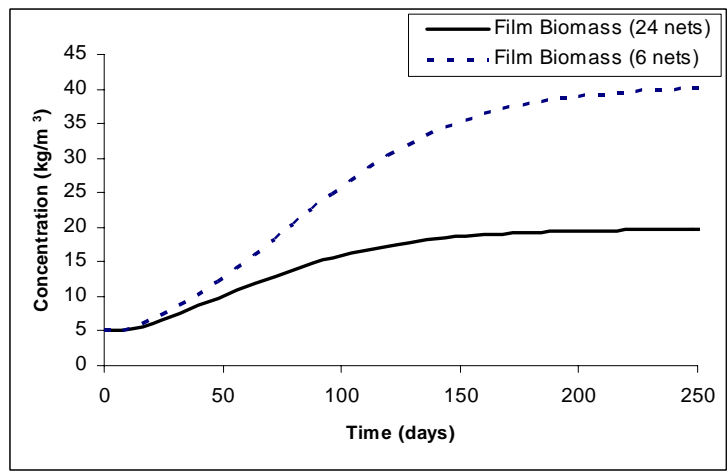


**Figure 9: Dynamic profile of attached biomass with 24 nets**



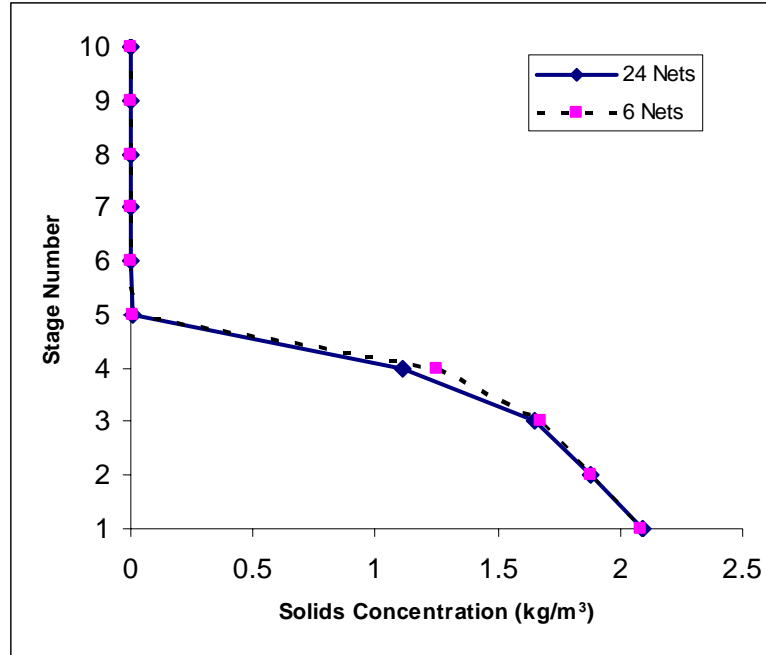
**Figure 10: Dynamic profile of solid concentration in the clarifier for 24 nets**

Figure 11 compares the dynamics of film biomass concentration for 6 and 24 nets. It is seen that the final biomass concentration is higher for the case with 6 nets.



**Figure 11: Dynamic profiles of attached biomass concentrations**

Figure 12 shows the steady state profile of the solids concentration along the stages (layers) of the clarifier for both the cases. The solids concentrations at the bottom and top layers are almost the same.



**Figure 12: Steady state solids concentration profiles**

## **6. Conclusions:**

Based on the data available for the wastewater Hybrid Systems, a heterogeneous is developed and verified against such data. The results showed that the model results correlate well, but are slightly less than the experimental values. It is observed that the experimental and model results generally exhibit the similar trend between different cases. But, the effluent BOD ( $S_E$ ) calculated from the model does not change with the change in the number of nets, while  $S_E$  drops with an increase in the number of nets in the experiment. More experimental data is needed for better fitting and fine tuning of the model parameters. Pilot plant data from the hybrid growth WWTP in Zenin, Egypt is expected in near future, which will be used for improving the model.

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